

B.Sc. Semester - 3 (NEP-2020) Examination
OCT/NOV-2025 (AS PER SYLLABUS YEAR JUNE 2025)
MATH: Advanced Calculus -1 (Major-6)

Time: 2:00 Hours

Marks:50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two out of the following. (10)

1. Define: Scalar point function.

Prove: i) $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$$\text{ii) } \nabla\left(\frac{\phi}{\psi}\right) = \frac{\psi\nabla\phi - \phi\nabla\psi}{\psi^2}$$

2. Prove: $\text{div}(\bar{f} \times \bar{g}) = \bar{g} \text{curl } \bar{f} - \bar{f} \cdot \text{curl } \bar{g}$.

3. If $\bar{f}$ and $\bar{g}$ are irrotational functions then show that $\bar{f} \times \bar{g}$ is solenoidal function.

Que.2 Answer any two out of the following. (10)

1. Find area of ellipse by double integral.

2. Show that $\iint_R (x-y)^2 \sin^2(x+y) dx dy = \frac{\pi^4}{3}$, where R is parallelogram whose vertices are $(\pi, 0), (2\pi, \pi), (\pi, 2\pi), (0, \pi)$.

3. Find: $\iiint_R xyz dx dy dz$, where R is $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

Que.3 Answer any two out of the following. (10)

1. Discuss the area of a region using Green's theorem and the find the area of ellipse using the Green's theorem.
2. Find the surface area of the sphere having radius a.
3. State and prove divergence theorem.

Que.4 Answer any two out of the following. (10)

1. Prove: $\beta(p, q) = 2 \int_0^{\frac{\pi}{2}} \sin^{2p-1} \theta \cos^{2q-1} \theta d\theta$.

2. Show that $\Gamma n = \int_0^\infty \left(\log \frac{1}{y}\right)^{n-1} dy$.

3. Prove that $\int_0^1 \frac{x^5}{\sqrt{1-x^4}} dx = \frac{\pi}{8}$

Que.5 Answer any two out of the following. (10)

1. State and prove the relation between Beta and Gamma function.
2. Verify stokes theorem for $\bar{F} = x^2\bar{i} + xy\bar{j}$ and S is a square whose sides are $x = 0, y = 0, x = a, y = a$ where $z = 0$.
3. If $\bar{f}$ is solenoidal function then find a, where
 $\bar{f} = (ax + 3y + 4z)\bar{i} + (x - 2y + 3z)\bar{j} + (3x + 2y - z)\bar{k}$.
